

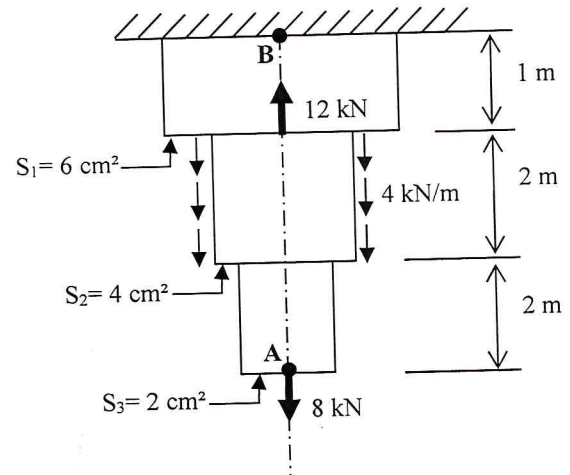
Examen : RDM-MS 431

Exercice 1 :

Pour le système ci-dessous, il est demandé de :

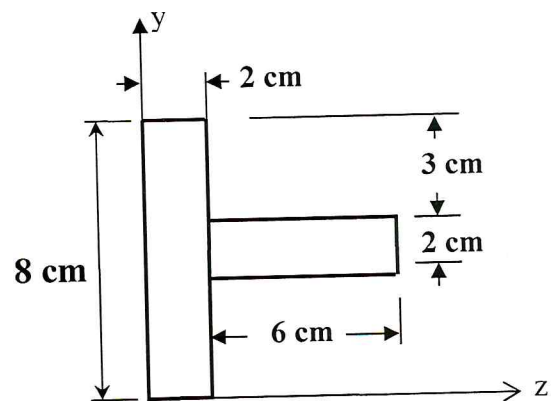
1. Calculer la réaction au point B,
2. Tracer le diagramme de l'effort normal $N(x)$, ainsi que celui de la contrainte normale $\sigma(x)$.
2. Déterminer le déplacement du point A,

On donne : $E = 21.10^3 \text{ daN/cm}^2$



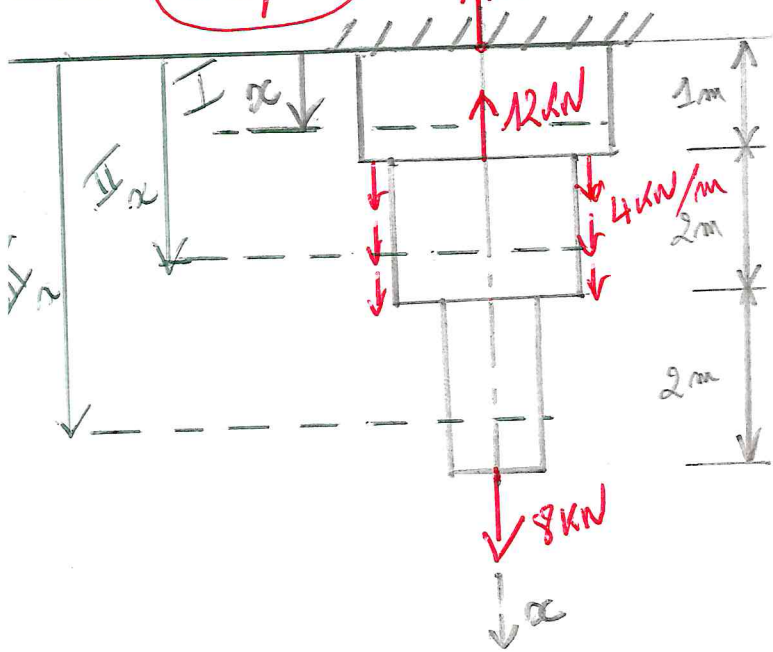
Exercice 2 :

Déterminer les **coordonnées** du centre de gravité (G) et les **moments d'inertie centraux et principaux** de la section présentée dans la figure.



CORRECTION EXAMEN RDM MS 431

EX1: 11 pts



1/ Calcul de R_e :

$$\sum \vec{F}_x = \vec{0} \Rightarrow 8 + 4 \times 2 - 12 - R_e = 0$$

$$\Rightarrow \boxed{R_e = 4 \text{ kN}} \quad \underline{1 \text{ pt}}$$

2/ Diagramme de $N(x)$ et $G(x)$:

1^{er} tronçon: $0 \leq x \leq 1 \text{ m}$

$$\sum \vec{F}_x = \vec{0} \Rightarrow N_1(x) - R_e = 0$$

$$\Rightarrow \boxed{N_1(x) = 4 \text{ kN}} \quad \underline{0,5 \text{ pt}}$$

$$G_1(x) = \frac{N_1(x)}{S_1} = \frac{4}{6 \cdot 10^{-4}}$$

$$\Rightarrow \boxed{G_1(x) = \frac{2}{3} \cdot 10^4 \text{ kN/m}^2} \quad \underline{0,5 \text{ pt}}$$

2^{ème} tronçon: $1 \leq x \leq 3 \text{ m}$:

$$\sum \vec{F}_x = \vec{0} \Rightarrow N_2(x) + 9(x-1) - 12 - R_e = 0$$

$$\Rightarrow N_2(x) = 12 + R_e - 9(x-1)$$

$$\Rightarrow \boxed{N_2(x) = -4x + 20} \quad \underline{0,5 \text{ pt}}$$

$$N_2(1) = +16 \text{ kN} \quad \underline{0,5 \text{ pt}}$$

$$N_2(3) = +8 \text{ kN}$$

$$G_2(x) = \frac{N_2(x)}{S_2} = \frac{-4x + 20}{4 \cdot 10^{-4}}$$

$$\boxed{G_2(x) = -10^4 x + 5 \cdot 10^4} \quad \underline{0,5 \text{ pt}}$$

$$G_2(1) = 4 \cdot 10^4 \text{ kN/m}^2 \quad \underline{0,5 \text{ pt}}$$

$$G_2(3) = 2 \cdot 10^4 \text{ kN/m}^2$$

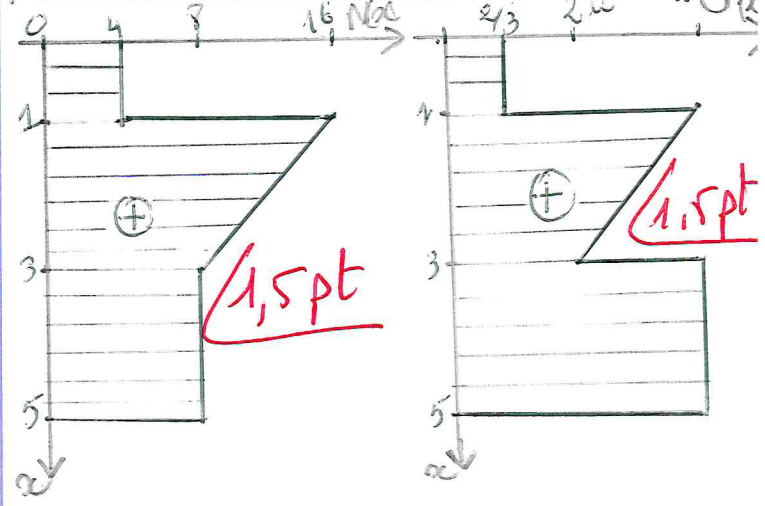
3^{ème} tronçon: $3 \leq x \leq 5 \text{ m}$

$$\sum \vec{F}_x = \vec{0} \Rightarrow N_3(x) + 9 \cdot 2 - 12 - R_e = 0$$

$$\Rightarrow \boxed{N_3(x) = +8 \text{ kN}} \quad \underline{0,5 \text{ pt}}$$

$$G_3(x) = \frac{N_3(x)}{S_3} = \frac{8}{2 \cdot 10^{-4}}$$

$$\boxed{G_3(x) = 4 \cdot 10^4 \text{ kN/m}^2} \quad \underline{0,5 \text{ pt}}$$



3/ Déplacement du pt "A"

$$\Delta l_T = \Delta l_1 + \Delta l_2 + \Delta l_3 \quad \underline{0,5 \text{ pt}}$$

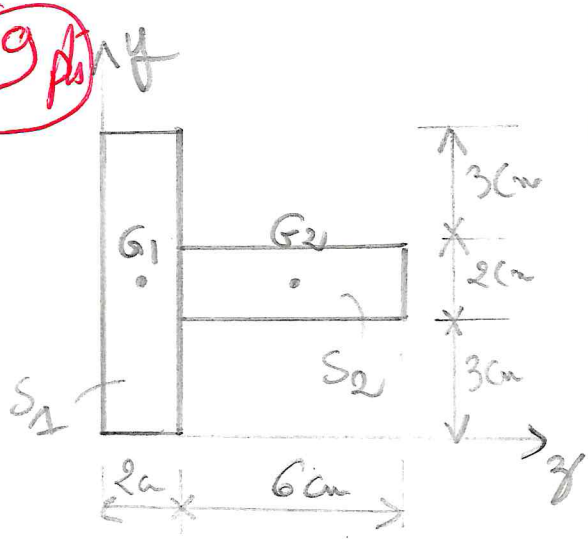
$$\Delta P_1 = \int_0^1 \frac{G_1(x)}{E} dx = 3,17 \cdot 10^{-3} \text{ m} \quad (0,5 \text{ pt})$$

$$\Delta P_2 = \int_1^3 \frac{G_2(x)}{E} dx = 28,57 \cdot 10^{-3} \text{ m} \quad (0,5 \text{ pt})$$

$$\Delta P_3 = \int_3^5 \frac{G_3(x)}{E} dx = 19,04 \cdot 10^{-3} \text{ m} \quad (0,5 \text{ pt})$$

$$\Delta P_T = 50,78 \cdot 10^{-3} \text{ m} \quad (1 \text{ pt})$$

Ex2: (09 pts)



Section 1:

$$S_1 = 2 \times 8 = 16 \text{ cm}^2 \quad (0,5 \text{ pt})$$

$$G_1(1, 4) \quad (0,5 \text{ pt})$$

$$I_{z_{G_1}} = \frac{bR^3}{12} = \frac{2 \cdot 8^3}{12} = 85,33 \text{ cm}^4 \quad (0,5 \text{ pt})$$

$$I_{y_{G_1}} = \frac{Rb^3}{12} = \frac{8 \cdot 2^3}{12} = 5,33 \text{ cm}^4 \quad (0,5 \text{ pt})$$

$$I_{y, z_1} = 0 \quad (0,25 \text{ pt})$$

Section 2:

$$S_2 = 2 \times 6 = 12 \text{ cm}^2 \quad (0,5 \text{ pt})$$

$$G_2(5, 4) \quad (0,5 \text{ pt})$$

$$I_{z_{G_2}} = \frac{bR^3}{12} = \frac{6 \cdot 2^3}{12} = 4 \text{ cm}^4 \quad (0,5 \text{ pt})$$

$$I_{y_{G_2}} = \frac{Rb^3}{12} = \frac{2 \cdot 6^3}{12} = 36 \text{ cm}^4 \quad (0,5 \text{ pt})$$

$$I_{y, z_2} = 0 \quad (0,25 \text{ pt})$$

* Centre de gravité "G"

$$z_G = \frac{\sum z_{G_i} S_i}{\sum S_i} = \frac{z_{G_1} S_1 + z_{G_2} S_2}{S_1 + S_2} \quad (0,25 \text{ pt})$$

$$z_G = 2,71 \text{ cm} \quad (0,5 \text{ pt})$$

$$y_G = \frac{\sum y_{G_i} S_i}{\sum S_i} = \frac{y_{G_1} S_1 + y_{G_2} S_2}{S_1 + S_2} \quad (0,25 \text{ pt})$$

$$y_G = 4 \text{ cm} \quad (0,5 \text{ pt})$$

$$G(2,71; 4)$$

* Les moments d'inertie centriques.

$$I_{z_G} = \sum I_{z_{G_i}} + a_i^2 S_i \quad (0,25 \text{ pt})$$

$$a_i = y_{G_i} - y_G$$

$$a_1 = y_{G_1} - y_G = 0 \quad (0,25 \text{ pt})$$

$$a_2 = y_{G_2} - y_G = 0$$

$$I_{z_G} = I_{z_{G_1}} + I_{z_{G_2}} + a_1^2 S_1 + a_2^2 S_2$$

$$= 85,33 + 4 + 0 + 0$$

$$I_{z_G} = 89,33 \text{ cm}^4 \quad (0,5 \text{ pt})$$

$$I_{y_G} = \sum I_{y_{G_i}} + b_i^2 S_i \quad (0,25 \text{ pt})$$

$$= I_{y_{G_1}} + I_{y_{G_2}} + b_1^2 S_1 + b_2^2 S_2$$

$$b_i = z'_{G_i} - z'_G$$

$$b_1 = z'_{G_1} - z'_G = -1,71 \text{ cm}$$

$$b_2 = z'_{G_2} - z'_G = 2,29 \text{ cm} \quad (0,25 \text{ pt})$$

$$I_{y_G} = 5,33 + 36 + (-1,71)^2 (16) + (2,29)^2 (12)$$

$$I_{y_G} = 151,04 \text{ cm}^4 \quad (0,5 \text{ pt})$$

$$I_{y_G z'_G} = \sum I_{y_{G_i} z'_{G_i}} + a_i b_i S_i \quad (0,25 \text{ pt})$$

$$= I_{y_1 z_1} + I_{y_2 z_2} + a_1 b_1 S_1$$

$$+ a_2 b_2 S_2$$

$$= 0 + 0 + 0 + 0$$

$$I_{y_G z'_G} = 0 \quad (0,25)$$

$I_{y_G z'_G} = 0 \Rightarrow$ Les axes centraux

z'_G et y'_G sont principaux

$(0,5 \text{ pt})$