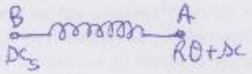
Solution Examen Final 2020/2021

Exercice 1

1/ $E_c = E_{c_{trans}} + E_{c_{rot}} = \frac{1}{2} M\dot{x}^2 + \frac{1}{2} J\dot{\theta}^2 = \frac{1}{2} M\dot{x}^2 + \frac{1}{2}\frac{1}{2} MR^2\dot{\theta}^2$

$\boxed{E_c = \frac{3}{4} M\dot{x}^2}$

$E_p = E_{p_k} = \frac{1}{2} k(2x - x_s)^2$



$D = \frac{1}{2}\beta\dot{x}^2$

2/ $L = E_c - E_p$; $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = -\frac{\partial D}{\partial \dot{x}}$

$\frac{3}{2} M\ddot{x} + 2k(2x - x_s) = -\beta\dot{x} \Rightarrow$

$\frac{3}{2} M\ddot{x} + \beta\dot{x} + 4kx = +2kx_s$

$\Rightarrow \boxed{\ddot{x} + \frac{2\beta}{3M}\dot{x} + \frac{8k}{3M}x = \frac{4k}{3M}S_0\cos\Omega t}$

L'eqt diff du H^{vt} avec : $2\gamma = \frac{2\beta}{3M}$, $\omega_0^2 = \frac{8k}{3M}$

3/ La solution du régime permanent est la solution particulière càd du m̂ type que le second mbre.

$x_p = A\cos(\Omega t + \Phi) = Ae^{i(\Omega t + \Phi)}$

$x_s = S_0 e^{i\Omega t}$

$\rightarrow \dot{x}_p = Ai\Omega\, e^{i(\Omega t + \Phi)}$ $\rightarrow \ddot{x}_p = -A\Omega^2 e^{i(\Omega t + \Phi)}$

On remplace dans l'équation, on aura :

$Ae^{i\varphi}\left((-\Omega^2 + \omega_0^2) + i2\gamma\Omega\right) = \frac{4k}{3M} S_0$

$\Rightarrow \underbrace{A(-\Omega^2 + \omega_0^2 + i\,2\gamma\Omega)}_{z_1} = \underbrace{\frac{4k}{3M} S_0 e^{-i\varphi}}_{z_2}$

$z_1 = z_2 \Rightarrow |z_1| = |z_2|$ et l'argument de z_1 = l'arg z_2.

$\Rightarrow A = \frac{\frac{4k}{3M} S_0}{\sqrt{(-\Omega^2 + \omega_0^2)^2 + 4\gamma^2\Omega^2}}$; $\Phi = -\mathrm{arctg}\,\frac{2\gamma\Omega}{-\Omega^2 + \omega_0^2}$

4/ L'expression de la pulsation de résonance :

$$A \nearrow \Rightarrow \frac{dA}{d\Omega} = 0 \Rightarrow \Omega_r = \sqrt{\omega_0^2 - 2\gamma^2} = \sqrt{\frac{8k}{3M} - \frac{2\beta^2}{9M^2}}$$

<u>Exercice 2</u>

1/ $E_c = E_{c_{m_1}} + E_{c_{m_2}} + E_{c_{m_3}} = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} J_1 \dot{\theta}^2 + \frac{1}{2} J_2 \dot{\theta}^2$

$$E_c = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 L^2 \dot{\theta}^2 + \frac{1}{2} m_3 L^2 \dot{\theta}^2 = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} (m_2 + m_3) L^2 \dot{\theta}^2$$

$$E_p = E_{p_{k_1}} + E_{p_{k_2}} + E_{p_{m_2}} + E_{p_{m_3}} + E_{p_{k_3}}$$

$$E_p = \frac{1}{2} k_1 x^2 + \frac{1}{2} k_2 (x - L\theta)^2 + \frac{1}{2} k_3 L^2 \theta^2 - m_2 g L \cos\theta + m_3 g L \cos\theta$$

$$D = \frac{1}{2} \beta L^2 \dot{\theta}^2$$

$$L = E_c - E_p \quad ; \quad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \underset{\dot{\theta}}{\dot{x}}}\right) - \frac{\partial \mathcal{L}}{\partial \underset{\theta}{x}} = - \frac{\partial D}{\partial \underset{\dot{\theta}}{\dot{x}}}$$

$$\Rightarrow \begin{cases} m_1 \ddot{x} + k_1 x + k_2 (x - L\theta) = 0 \\ (m_2 + m_3) L^2 \ddot{\theta} + k_2 L (x - L\theta) + k_3 L^2 \theta + (m_2 - m_3) g L \overbrace{\sin\theta}^{\theta} = -\beta L^2 \dot{\theta} \end{cases}$$

$$\Rightarrow \begin{cases} m_1 \ddot{x} + (k_1 + k_2) x - k_2 L \theta = \\ (m_2 + m_3) L^2 \ddot{\theta} + \beta L^2 \dot{\theta} + (k_2 L^2 + k_3 L^2 + (m_2 - m_3) g L) \theta - k_2 L x = 0 \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} \ddot{x} + \frac{k_1 + k_2}{m_1} x - \frac{k_2 L}{m_1} \theta = 0 \\ \ddot{\theta} + \frac{\beta}{m_2 + m_3} \dot{\theta} + \frac{(k_2 L^2 + k_3 L^2 + (m_2 - m_3) g L)}{(m_2 + m_3) L^2} \theta - \frac{k_2}{(m_2 + m_3) L} x = 0 \end{cases}}$$

2/ $\beta = 0$; $m_1 = 2m$; $m_2 = m_3 = m$; $k_1 = k_2 = k_3 = k$

$$\Rightarrow \begin{cases} \ddot{x} + \frac{k}{m} x - \frac{kL}{2m} \theta = 0 \\ \ddot{\theta} + \frac{k}{m} \theta - \frac{k}{2mL} x = 0 \end{cases}$$

Les solutions :

$$\begin{cases} x = A_1 \cos(\omega t + \varphi_1) \\ \theta = A_2 \cos(\omega t + \varphi_2) \end{cases}$$

$$\rightarrow \begin{cases} x = A_1 e^{i(\omega t + \varphi_1)} = \bar{A}_1 e^{i\omega t} & \text{avec } \bar{A}_1 = A_1 e^{i\varphi_1} \\ \theta = A_2 e^{i(\omega t + \varphi_2)} = \bar{A}_2 e^{i\omega t} & \text{"} \quad \bar{A}_2 = A_2 e^{i\varphi_2} \end{cases}$$