Examen final (Corrigé type)

Exo 1 :

1) $$\iint_{D_1} x^2 y \, dx\, dy = \int_0^2 \int_0^{x^2} x^2 y \, dx\, dy = \int_0^2 \left[\int_0^{x^2} x^2 y \, dy \right] dx$$

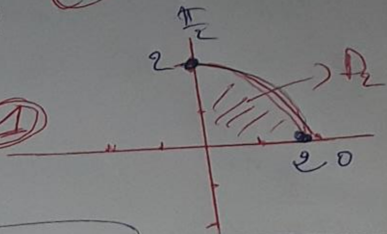
$$= \int_0^2 \left[\frac{1}{2} x^2 y^2 \right]_0^{x^2} dx = \frac{1}{2} \int_0^2 x^6 \, dx = \frac{1}{2} \left[\frac{1}{7} x^7 \right]_0^2$$

1

$$= \frac{2^7}{14} = \frac{128}{14} = \frac{64}{7}$$

1

2) a)



b) On pose $\overset{0{,}25}{\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}} \Rightarrow \begin{cases} r^2 = x^2 + y^2 \\ \underbrace{dx\,dy = r\,dr\,d\theta}_{0{,}25} \end{cases}$

$\underbrace{0 \leq r \leq 2}_{0{,}25}$, $\underbrace{0 \leq \theta \leq \frac{\pi}{2}}_{0{,}25}$.

Donc
$$\iint_{D_2} \sqrt{x^2+y^2} \, dx\, dy = \int_0^2 \int_0^{\frac{\pi}{2}} \sqrt{r^2} \, r \, d\theta \, dr$$

$$= \int_0^2 r^2 \, dr \int_0^{\frac{\pi}{2}} d\theta = \left[\frac{1}{3} r^3 \right]_0^2 \cdot \left[\theta \right]_0^{\frac{\pi}{2}}$$

1

$$= \frac{8}{3} \cdot \frac{\pi}{2} = \frac{4}{3}\pi$$

Exo 2 ; 0,5 0,5 (conclusion)

$$\lim_{n \to +\infty} \frac{n^2+1}{n^2+3} = 1 \neq 0$$ div 0,5

$$\lim_{n \to +\infty} e^{\frac{1}{n^2+1}} = 1 \neq 0$$ div

0,5 0,5 0,5

Exo 3 :

$$\frac{e^{-n^2}}{1+n^2} < \frac{1}{1+n^2} \underset{+\infty}{\approx} \frac{1}{n^2}$$ 1

$\sum \frac{1}{n^2}$ série de Rieman conv. => Conv

0,5

$$\sum \frac{x^n}{n!} \quad , x \in \mathbb{R}, \; n \in \mathbb{N}$$

$\frac{x^n}{n!}$ change le signe, donc on prend la valeur absolue. $\sum \left| \frac{x^n}{n!} \right|$

15

on applique d'Alembert, on pose $U_n = \left| \frac{x^n}{n!} \right|$

$$\lim_{n \to +\infty} \frac{U_{n+1}}{U_n} = \lim_{n \to +\infty} \left| \frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}} \right| = \lim_{n \to +\infty} \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n}$$

on sait que $(n+1)! = (n+1)n!$

donc $$\lim_{n \to +\infty} \frac{U_{n+1}}{U_n} = \lim_{n \to +\infty} \frac{x^{n+1}}{(n+1)n!} \cdot \frac{n!}{\cancel{x^n}} = \lim_{n \to +\infty} \frac{x}{n+1} = 0$$

$= 0 < 1 \Rightarrow$ convergence