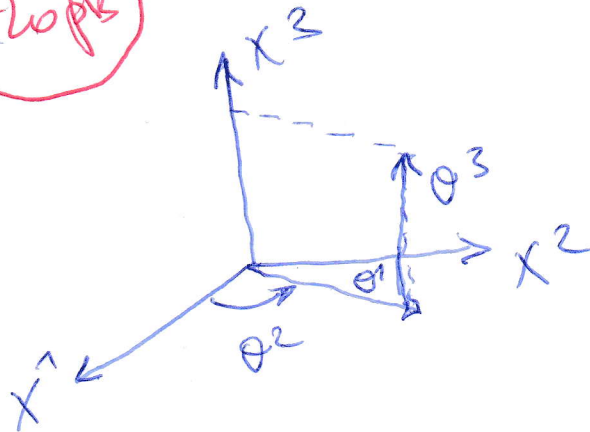


MCC

Exercice (20pts)



1/ (07,50pts)

$$\begin{aligned} x^1 &= \theta^1 \cos \theta^2 \\ x^2 &= \theta^1 \sin \theta^2 \\ x^3 &= \theta^3 \end{aligned}$$

0,5x2=1. $g_{ij} = \frac{\partial x^k}{\partial \theta^i} \frac{\partial x^k}{\partial \theta^j} \Rightarrow g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & (\theta^1)^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

0,5x2=1 $g^{ik} g_{kj} = \delta_j^i \Rightarrow g^{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{(\theta^1)^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

0,25 $\Gamma_{ijr} = \frac{1}{2} \left(\frac{\partial g_{ir}}{\partial \theta^j} + \frac{\partial g_{jr}}{\partial \theta^i} - \frac{\partial g_{ij}}{\partial \theta^r} \right)$

0,5x4=2. $\Rightarrow \Gamma_{12} = \theta^1, \Gamma_{21} = \theta^1, \Gamma_{22} = -\theta^1$ (les autres sont zéros)

0,25 $\Gamma_{ij}^k = g^{kr} \Gamma_{ijr}$

0,5x4=2. $\Rightarrow \Gamma_{12}^2 = \frac{1}{\theta^1}, \Gamma_{21}^2 = \frac{1}{\theta^1}, \Gamma_{22}^1 = -\theta^1$ (les autres sont zéros)

$$2) \textcircled{\partial \tau_{ij}^k} \tau_{ij}^k + f^i = 0$$

$$0,25 \quad \tau_{ij}^k = \frac{\partial \tau_{ij}^k}{\partial \theta^k} + \tau_{ik}^a \Gamma_{ak}^i + \tau_{kj}^a \Gamma_{ak}^j$$

$$\Rightarrow \left. \begin{array}{l} \tau_{11}^1 + \tau_{12}^2 + \tau_{13}^3 + f^1 = 0 \\ \tau_{12}^1 + \tau_{22}^2 + \tau_{23}^3 + f^2 = 0 \\ \tau_{13}^1 + \tau_{23}^2 + \tau_{33}^3 + f^3 = 0 \end{array} \right\} \begin{array}{l} 3 \times 0,25 \\ = 0,75 \end{array}$$

$$\text{avec: } \tau_{11}^1 = \frac{\partial \tau_{11}^1}{\partial \theta^1}$$

$$\tau_{12}^1 = \frac{\partial \tau_{12}^1}{\partial \theta^2} + \frac{1}{\theta^1} \tau_{11}^1 - \theta^1 \tau_{22}^2$$

$$\tau_{13}^1 = \frac{\partial \tau_{13}^1}{\partial \theta^3}$$

$$\tau_{12}^2 = \frac{\partial \tau_{12}^2}{\partial \theta^1} + \frac{1}{\theta^1} \tau_{12}^2$$

$$\tau_{22}^2 = \frac{\partial \tau_{22}^2}{\partial \theta^2} + \frac{1}{\theta^1} \tau_{12}^2 + \frac{1}{\theta^1} \tau_{21}^1$$

$$\tau_{23}^2 = \frac{\partial \tau_{23}^2}{\partial \theta^3}$$

$$\tau_{13}^3 = \frac{\partial \tau_{13}^3}{\partial \theta^1}$$

$$\tau_{23}^3 = \frac{\partial \tau_{23}^3}{\partial \theta^2}$$

$$\tau_{33}^3 = \frac{\partial \tau_{33}^3}{\partial \theta^3}$$

$$9 \times 0,15 \\ = 1,35$$

2x05
= 115
=>

$$\frac{\partial \tau^{11}}{\partial \theta^1} + \frac{1}{\theta^1} \tau^{11} + \frac{\partial \tau^{21}}{\partial \theta^2} - \theta^1 \tau^{22} + \frac{\partial \tau^{31}}{\partial \theta^3} + f^1 = 0$$

$$\frac{\partial \tau^{12}}{\partial \theta^1} + \frac{2}{\theta^1} \tau^{12} + \frac{1}{\theta^1} \tau^{21} + \frac{\partial \tau^{22}}{\partial \theta^2} + \frac{\partial \tau^{32}}{\partial \theta^3} + f^2 = 0$$

$$\frac{\partial \tau^{13}}{\partial \theta^1} + \frac{\partial \tau^{23}}{\partial \theta^2} + \frac{\partial \tau^{33}}{\partial \theta^3} + f^3 = 0$$

3) 05, 50p3 $\tau^{(i)} = \sqrt{\frac{g_{ij}}{g_{ii}}}$ (pas de somme)

0,5 $f^{(i)} = f^i \sqrt{g_{ii}}$ (pas de somme)

$$\tau^{(11)} = \tau^{11} \rightarrow \tau^{11} = \tau^{(11)}$$

$$\tau^{(12)} = \theta^1 \tau^{12} \rightarrow \tau^{12} = \frac{1}{\theta^1} \tau^{(12)}$$

$$\tau^{(13)} = \tau^{13} \rightarrow \tau^{13} = \tau^{(13)}$$

$$\tau^{(21)} = \theta^1 \tau^{21} \rightarrow \tau^{21} = \frac{1}{\theta^1} \tau^{(21)}$$

$$\tau^{(22)} = (\theta^1)^2 \tau^{22} \rightarrow \tau^{22} = \frac{1}{(\theta^1)^2} \tau^{(22)}$$

$$\tau^{(23)} = \theta^1 \tau^{23} \rightarrow \tau^{23} = \frac{1}{\theta^1} \tau^{(23)}$$

$$\tau^{(31)} = \tau^{31} \rightarrow \tau^{31} = \tau^{(31)}$$

$$\tau^{(32)} = \theta^1 \tau^{32} \rightarrow \tau^{32} = \frac{1}{\theta^1} \tau^{(32)}$$

$$\tau^{(33)} = \tau^{33} \rightarrow \tau^{33} = \tau^{(33)}$$

$$f^{(1)} = f^1 \rightarrow f^1 = f^{(1)}$$

$$f^{(2)} = \theta^1 f^2 \rightarrow f^2 = \frac{1}{\theta^1} f^{(2)}$$

$$f^{(3)} = f^3 \rightarrow f^3 = f^{(3)}$$

19x05
= 3