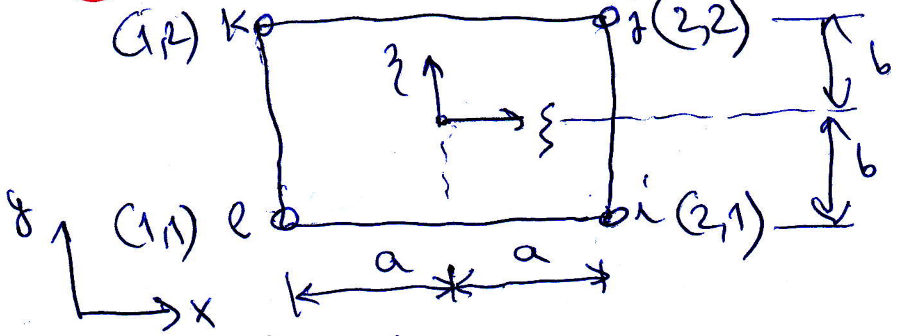
Houmat A. Examen MEF 2019/2020

Solutions

Ex 1 : 06



1/2

1/2 $a = 1/2$, $b = 1/2$

1 $\begin{cases} \bar{x} = \dfrac{x_j + x_k}{2} = \dfrac{x_i + x_l}{2} = \dfrac{2+1}{2} = 3/2 \\ \bar{y} = \dfrac{y_i + y_j}{2} = \dfrac{y_l + y_k}{2} = \dfrac{1+2}{2} = \dfrac{3}{2} \end{cases}$

1 $\begin{cases} x = a\xi + \bar{x} = \dfrac{1}{2}\xi + \dfrac{3}{2} = \dfrac{1}{2}(\xi + 3) \\ y = b\eta + \bar{y} = \dfrac{1}{2}\eta + \dfrac{3}{2} = \dfrac{1}{2}(\eta + 3) \end{cases}$

1/2 $dx = \frac{1}{2} d\xi$, $dy = \frac{1}{2} d\eta$

2.5

$$I = \int_{A^e} xy \, dA = \int_{-1}^{1}\int_{-1}^{1} \frac{1}{2}(\xi+3)\frac{1}{2}(\eta+3)\frac{1}{2}d\xi \frac{1}{2}d\eta$$

$$= \frac{1}{16}\int_{-1}^{1}(\xi+3)d\xi \int_{-1}^{1}(\eta+3)\,d\eta$$

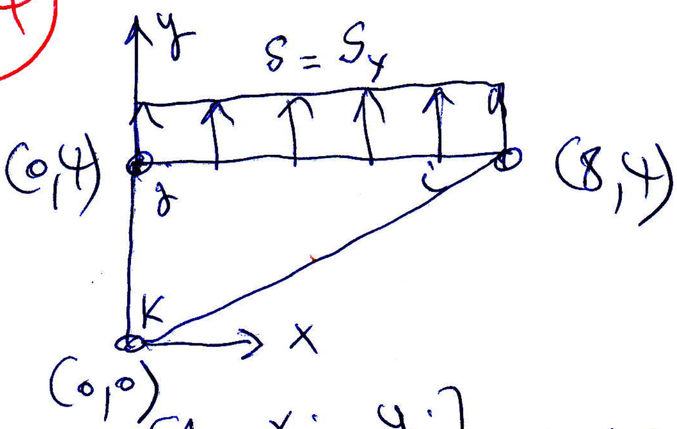
$$= \frac{1}{16}\left[\frac{1}{2}\xi^2 + 3\xi\right]_{-1}^{1}\left[\frac{1}{2}\eta^2 + 3\eta\right]_{-1}^{1}$$

$$= \frac{1}{16}(6)(6)$$

$$= \frac{9}{4}$$

Ex. 2. 14

1) 06



1/2

1/2 $A = \frac{1}{2}\det\begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} = \frac{1}{2}\det\begin{bmatrix} 1 & 8 & 4 \\ 1 & 0 & 4 \\ 1 & 0 & 0 \end{bmatrix} = 16\,cm^2$

1/4 $m_{21} = \frac{y_j - y_k}{2A} = \frac{4-0}{2\times16} = 0{,}125\ cm^{-1}$

1/4 $m_{31} = \frac{x_k - y_j}{2A} = \frac{0-0}{2\times16} = 0$

1/4 $m_{22} = \frac{y_k - y_i}{2A} = \frac{0-4}{2\times16} = -0{,}125\ cm^{-1}$

1/4 $m_{32} = \frac{x_i - x_k}{2A} = \frac{8-4}{2\times16} = 0{,}25\ cm^{-1}$

1/4 $m_{23} = \frac{y_i - y_j}{2A} = \frac{0-0}{2\times16} = 0$

1/4 $m_{33} = \frac{x_j - x_i}{2A} = \frac{4-8}{2\times16} = -0{,}25\ cm^{-1}$

1/4 $[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} = \frac{30\times10^6}{1-(0{,}3)^2}\begin{bmatrix} 1 & 0{,}3 & 0 \\ 0{,}3 & 1 & 0 \\ 0 & 0 & \frac{1-0{,}3}{2} \end{bmatrix}$

1 $= \begin{bmatrix} 33 & 9{,}89 & 0 \\ 9{,}89 & 33 & 0 \\ 0 & 0 & 11{,}5 \end{bmatrix}\times10^6\ N/cm^2$

1/4 $[K^e] = [B]^T[D][B]\,h\,A \qquad (h = 0{,}5\ cm)$

1 $[B] = \begin{bmatrix} 0{,}125 & 0 & -0{,}125 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0{,}25 & 0 & -0{,}25 \\ 0 & 0{,}125 & 0{,}25 & -0{,}125 & -0{,}25 & 0 \end{bmatrix}$

1 $$[k^e] = \begin{bmatrix} 4{,}12 & 0 & -4{,}12 & 2{,}48 & 0 & -2{,}48 \\ & 1{,}44 & 2{,}88 & -1{,}44 & -2{,}88 & 0 \\ & & 9{,}88 & -5{,}36 & -5{,}76 & 2{,}48 \\ & & & 17{,}94 & 2{,}88 & -17{,}50 \\ & \text{Sym.} & & & 5{,}76 & 0 \\ & & & & & 17{,}50 \end{bmatrix} \times 10^6 \ \text{N/cm}$$

2) 02

1 $$\{f^e\} = \{f_s^e\} = \frac{h\, l_{ij}}{2} \begin{Bmatrix} S_x \\ S_y \\ S_x \\ S_y \\ 0 \\ 0 \end{Bmatrix} = \frac{0{,}5 \times 8}{2} \begin{Bmatrix} 0 \\ 1000 \\ 0 \\ 1000 \\ 0 \\ 0 \end{Bmatrix}$$

1 $$= \begin{Bmatrix} 0 \\ 2000 \\ 0 \\ 2000 \\ 0 \\ 0 \end{Bmatrix} \ \text{N}$$

3) 06

1 $$[k^a]\{a^a\} = \{f^a\}$$

1 Conditions aux limites : $u_j = v_j = u_k = v_k = 0$

2 $$\begin{bmatrix} 4{,}12 & 0 & -4{,}12 & 2{,}48 & 0 & -2{,}48 \\ 0 & 1{,}44 & 2{,}88 & -1{,}44 & -2{,}88 & 0 \\ -4{,}12 & 2{,}88 & 9{,}88 & -5{,}36 & -5{,}76 & 2{,}48 \\ 2{,}48 & -1{,}44 & -5{,}36 & 17{,}94 & 2{,}88 & -17{,}50 \\ 0 & -2{,}88 & -5{,}76 & 2{,}88 & 5{,}76 & 0 \\ -2{,}48 & 0 & 2{,}48 & -17{,}50 & 0 & 17{,}50 \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2000 \\ 0 \\ 2000 \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \quad 10^6 \begin{bmatrix} 4{,}112 & 0 \\ 0 & 1{,}44 \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2000 \end{Bmatrix}$$

$$\Rightarrow \quad u_i = 0 \quad , \quad v_i = 0{,}0039 \text{ cm}$$